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# An extended study on steady-state laminar film condensation of a superheated vapour on an isothermal vertical plate

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**Abstract**—The dimensionless velocity component method is applied to transform the governing equations and its boundary conditions, instead of the traditional Falkner–Skan transformation. A temperature parameter method and a polynomial approach are used to treat variable thermophysical properties both for vapour and liquid films, respectively. The new dimensionless system of equations is computed numerically in two steps: first, shear force at the liquid–vapour interface is neglected, so that the initial values of $W_{xl,s}$ and $W_{yl,s}$ are obtained; and then, the calculation for a three-point boundary value problem coupling the equations of a liquid film with those of a vapour film are carried out. According to the numerical solutions and with a curving matching method, the corresponding simple and practical correlations of heat transfer coefficient and mass flow rate are developed by means of the superheated temperature $\Delta t_{\infty}$ of vapour and the subcooled temperature $\Delta t_w$ of a plate besides the defined local Grashof number of liquid film, $Gr_{xl,s}$.


## 1. INTRODUCTION

Since the pioneering work of Nusselt [1] in 1916 in treating laminar film condensation of saturated steam on a vertical isothermal flat plate, a number of studies have been done for successive investigations ignoring variable thermophysical properties [2–6] and with consideration of the variable thermophysical properties [7–11]. However, studies of the condensation of superheated vapour are scarcely found in the literature. Minkowycz and Sparrow reported their study results for condensation heat transfer with consideration of superheated vapour [12], and showed that superheated temperature brings only a slight increase in the heat transfer during the condensation of a pure vapour. They also indicated that for a given degree of superheating, $q/q_{Nu}$ is almost independent of $\Delta t_w$. However, so far, no theoretically quantitative correlation has been developed for the prediction of heat transfer, and especially, the effect of superheated temperature on mass transfer of the condensation in literature. The reason is that it is difficult to study the two-phase boundary layer problem, and that traditional theoretical methods for this kind of study, such as Falkner–Skan transformation for the similarity transformation of the governing partial differential equations and the method for treatment of variable thermophysical properties, are not suitable for the successive studies. Shang and Adamek have reported their study on film condensation of saturated vapour [13].

In this present work, we applied a set of advanced methods developed in our previous studies [14, 15], such as the dimensionless velocity component method and a polynomial approach, for similarity transformation of the governing partial differential equations of the two-phase boundary layer with consideration of various physical factors of liquid medium, and the polynomial method to treat variable thermophysical properties of the liquid medium. Consequently, we not only obtained practically simple correlations for predicting the heat transfer and mass flow rate for the condensation of saturated steam, but also investigated further the effect of superheated temperature on heat and mass transfer of laminar film condensation of superheated vapour on a vertical plate.

## 2. GOVERNING EQUATIONS FOR THE TWO-PHASE BOUNDARY LAYER

The analytical model and coordinating system used for laminar film condensation of superheated steam on a vertical flat plate is shown in Fig. 1. An isothermal vertical flat plate is suspended in a large volume of

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## NOMENCLATURE

- $c_p$ specific heat at constant pressure [kJ kg$^{-1}$ K$^{-1}$]
- $g$ gravitation acceleration [m s$^{-2}$]
- $Gr_{xl,s}$ local Grashof number of liquid film $[g(\rho_{l,w}-\rho_{v,\infty})x^3/\nu_{l,s}^2\rho_{l,s}]$
- $Gr_{xv,s}$ local Grashof number of vapour film $[g(\rho_{v,s}-\rho_{v,\infty})x^3/\nu_{v,\infty}^2\rho_{v,\infty}]$
- $h_{fg}$ latent heat of condensation [kJ kg$^{-1}$]
- $Nu_{xl,w}$ local Nusselt number, $\alpha_x x/\lambda_{l,w}$
- $Pr$ Prandtl number
- $q_{Nu}$ local heat transfer predicted by Nusselt theory
- $q_x$ local heat transfer rate per unit area from wall to liquid [W m$^{-2}$]
- $t$ temperature [°C]
- $t_s$ saturated temperature [°C]
- $T$ absolute temperature [K]
- $w_x$, $w_y$ velocity components in the $x$- and $y$-direction [m s$^{-1}$]
- $W_x$, $W_y$ dimensionless velocity components in the $x$- and $y$-direction
- $W_{xl,s}$, $W_{yl,s}$ dimensionless velocity components of liquid medium at the liquid–vapour interface in the $x$- and $y$-direction.

Greek symbols

- $\alpha_x$ local heat transfer coefficient [W m$^{-2}$ K$^{-1}$]
- $\delta$ boundary layer thickness [m]
- $\delta_l$ boundary layer thickness of liquid film [m]
- $\eta$ dimensionless co-ordinate variable for boundary layer
- $\eta_{\delta_l}$ dimensionless boundary layer thickness of liquid film
- $\theta$ dimensionless temperature
- $\rho$ density [kg m$^{-3}$]
- $\lambda$ thermal conductivity [W m$^{-1}$ K$^{-1}$]
- $\mu$ absolute viscosity [kg m$^{-1}$ s$^{-1}$]
- $\nu$ kinematic viscosity [m$^2$ s$^{-1}$]
- $\Delta t_w$ subcooled temperature on surface $t_s - t_w$ [°C]
- $\Delta t_\infty$ superheated vapour temperature, $t_\infty - t_s$ [°C]
- $\eta_{l\delta} W_{xl,s} - 4W_{yl,s}$ mass flow rate parameter of film condensation
- $(dW_{xl}/d\eta_l)_{\eta_l=\eta_{l,0}}$ dimensionless temperature gradient on the wall for film condensation.

Subscripts

- l liquid
- s saturate state
- v vapour
- w wall
- $\infty$ far from the wall surface.

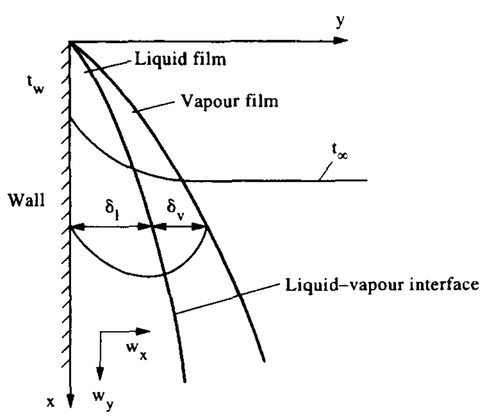


Fig. 1. Physical model and coordinate system.

quiescent pure superheated vapour at atmospheric pressure. The plate temperature is $t_w$, the saturation temperature of the vapour is $t_s$, and the ambient temperature is $t_\infty$.

If the condition for the model is $t_w < t_s$ and $t_s < t_\infty$, steady two-dimensional (2D) film condensation will occur on the plate. We assume that laminar flow within the liquid and vapour phases is induced by gravity, and take into account the various physical factors including variable thermophysical properties of the medium in the condensate and vapour film. Then the conservation governing equations of mass, momentum and energy for steady laminar condensation in two-phase boundary layer can be written as follows:

for liquid film:

$$\frac{\partial}{\partial x}(\rho_l w_{xl}) + \frac{\partial}{\partial y}(\rho_l w_{yl}) = 0 \tag{1}$$

$$\rho_l\left(w_{xl}\frac{\partial w_{xl}}{\partial x} + w_{yl}\frac{\partial w_{xl}}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu_l \frac{\partial w_{xv}}{\partial y}\right) + g(\rho_l - \rho_{v,\infty}) \tag{2}$$

$$\rho_l c_{pl}\left(w_{xl}\frac{\partial t_l}{\partial x} + w_{yl}\frac{\partial t_l}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda_l \frac{\partial t_l}{\partial y}\right) \tag{3}$$

for vapour film:

$$\frac{\partial}{\partial x}(\rho_v w_{xv}) + \frac{\partial}{\partial y}(\rho_v w_{yv}) = 0 \tag{4}$$

$$\rho_v\left(w_{xv}\frac{\partial w_{xv}}{\partial x}+w_{yv}\frac{\partial w_{xv}}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu_v\frac{\partial w_{xv}}{\partial y}\right)+g(\rho_v-\rho_{v,\infty}) \quad (5)$$

$$\rho_v c_{pv}\left(w_{xv}\frac{\partial t_v}{\partial x}+w_{yv}\frac{\partial t_v}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_v\frac{\partial t_v}{\partial y}\right) \quad (6)$$

with boundary conditions:

$$y=0: \quad w_{xl}=0 \quad w_{yl}=0 \quad t_l=t_w \quad (7)$$

$$y=\delta_l: \quad w_{xl,s}=w_{xv,s} \quad (8)$$

$$\rho_{l,s}\left(w_{xl}\frac{\partial \delta_{xl}}{\partial x}-w_{yl}\right)_s=\rho_{v,s}\left(w_{xv}\frac{\partial \delta_{xv}}{\partial x}-w_{yv}\right)_s \quad (9)$$

$$\mu_{l,s}\left(\frac{\partial w_{xl}}{\partial y}\right)_s=\mu_{v,s}\left(\frac{\partial w_{xv}}{\partial y}\right)_s \quad (10)$$

$$-\lambda_{l,s}\left(\frac{\partial t_l}{\partial y}\right)_{y=\delta_l}=h_{fg}\rho_{l,s}\left(w_{xl}\frac{\partial \delta_l}{\partial x}-w_{yl}\right)_s-\lambda_{v,s}\left(\frac{\partial t_v}{\partial y}\right)_{y=d_l} \quad (11)$$

$$t=t_s \quad (12)$$

$$y\to\infty: \quad w_{xv}\to 0 \quad t_v\to t_\infty. \quad (13)$$

Equations (8)–(11) express the physical matching conditions such as the continuities of velocity, local mass flux, shear force and heat flux at the vapour–liquid interface, respectively.

## 3. SIMILARITY TRANSFORMATION

Using the dimensionless velocity component method developed in [14, 15], we assume the following dimensionless variables for the similarity transformation of the governing partial differential equations of condensation.

for liquid film:

$$\eta_l=\left(\frac{1}{4}Gr_{xl,s}\right)^{1/4}\frac{y}{x} \quad Gr_{xl,s}=\frac{g(\rho_{l,w}-\rho_{v,\infty})x^3}{v_{l,s}^2\rho_{l,s}} \quad (14)$$

$$\theta_l=\frac{t-t_\infty}{t_w-t_\infty} \quad (15)$$

$$W_{xl}=\left(2\sqrt{gx}\left(\frac{\rho_{l,w}-\rho_{v,\infty}}{\rho_{l,s}}\right)^{1/2}\right)^{-1}w_{xl} \quad (16)$$

$$W_{yl}=\left(2\sqrt{gx}\left(\frac{\rho_{l,w}-\rho_{v,\infty}}{\rho_{l,s}}\right)^{1/2}\left(\frac{1}{4}Gr_{xl,s}\right)^{-4}\right)^{-1}W_{yl}. \quad (17)$$

for vapour film:

$$\eta_v=\left(\frac{1}{4}Gr_{xv,\infty}\right)^{1/4}\frac{y}{x} \quad Gr_{xv,\infty}=\frac{g(\rho_{v,s}/\rho_{v,\infty}-1)x^3}{v_{v,\infty}^2} \quad (18)$$

$$\theta_v=\frac{t-t_\infty}{t_s-t_\infty} \quad (19)$$

$$W_{xv}=(2\sqrt{gx}(\rho_{v,s}/\rho_{v,\infty}-1)^{1/2})^{-1}w_{xv} \quad (20)$$

$$W_{yv}=(2\sqrt{gx}(\rho_{v,s}/\rho_{v,\infty}-1)^{1/2}(\tfrac{1}{4}Gr_{xv,\infty})^{-1/4})^{-1}w_{yv}. \quad (21)$$

Then, the following ordinary differential equations for the two-phase boundary layer and their boundary conditions are obtained:

for liquid film:

Equations (1)–(3) are transformed into dimensionless ordinary ones, respectively:

$$2W_{xl}-\eta_l\frac{dW_{xl}}{d\eta_l}+4\frac{dW_{yl}}{d\eta_l}+\frac{1}{\rho_l}\frac{d\rho_l}{d\eta_l}(-\eta_l W_{xl}+4W_{yl})=0 \quad (22)$$

$$\frac{v_{l,s}}{v_l}\left(W_{xl}\left(2W_{xl}-\eta_l\frac{dW_{xl}}{d\eta_l}\right)+4W_{yl}\frac{dW_{xl}}{d\eta_l}\right)=\frac{d^2W_{xl}}{d\eta_l^2}+\frac{1}{\mu_l}\frac{d\mu_l}{d\eta_l}\frac{dW_{xl}}{d\eta_l}+\frac{\mu_{l,s}}{\mu_l}\frac{\rho_l-\rho_{v,\infty}}{\rho_{l,w}-\rho_{v,\infty}} \quad (23)$$

$$Pr_l\frac{\rho_l}{\rho_{l,s}}\frac{\mu_{l,s}}{\mu_l}(-\eta_l W_{xl}+4W_{yl})\frac{d\theta_l}{d\eta_l}=\frac{d^2\theta_l}{d\eta_l^2}+\frac{1}{\lambda_l}\frac{d\lambda_l}{d\eta_l}\frac{d\theta_l}{d\eta_l}. \quad (24)$$

As reported in ref. [13], specific heat $c_{p_l}$ can be taken to be $c_{p_{l,s}}$ with a maximal deviation of only 0.455% for water. Hence, the factor

$$Pr_l\frac{\rho_l}{\rho_{l,s}}\frac{\lambda_{l,s}}{\lambda_l}$$

in equation (24) can be substituted by

$$Pr_{l,s}\frac{\rho_l}{\rho_{l,s}}\frac{\lambda_{l,s}}{\lambda_l}$$

accordingly.

for vapour film:

Equations (4)–(6) are transformed into dimensionless ordinary ones as

$$2W_{xv}-\eta_v\frac{dW_{xv}}{d\eta_v}+4\frac{dW_{yv}}{d\eta_v}+\frac{1}{\rho_v}\frac{d\rho_v}{d\eta_v}(-\eta_v W_{xv}+4W_{yv})=0 \quad (25)$$

$$\frac{v_{v,\infty}}{v_v}\left(W_{xv}\left(2W_{xv}-\eta_v\frac{dW_{xv}}{d\eta_v}\right)+4W_{yv}\left(\frac{dW_{xv}}{d\eta_v}\right)\right)=\frac{d^2W_{xv}}{d\eta_v^2}+\frac{1}{\mu_v}\frac{d\mu_v}{d\eta_v}\frac{dW_{xv}}{d\eta_v}+\frac{\mu_{v,\infty}}{\mu_v}\frac{\rho_v-\rho_{v,\infty}}{\rho_{v,s}-\rho_{v,\infty}} \quad (26)$$

$$Pr_v \frac{\nu_{v,\infty}}{\nu_v}(-\eta_v W_{xv}-4W_{yv})\frac{d\theta_v}{d\eta_v} = \frac{d^2\theta_v}{d\eta_v^2}+\frac{1}{\lambda_v}\frac{d\lambda_v}{d\eta_v}\frac{d\theta_v}{d\eta_v}. \tag{27}$$

for boundary conditions:

The boundary conditions, equations (7)–(13), are transformed to the following ones, respectively:

$$\eta_l = 0: \quad W_{xl}=0 \quad W_{yl}=0 \quad \theta_l = 1 \tag{28}$$

$$\eta_l = \eta_{l\delta}(\eta_v = 0):$$

$$W_{xv,s} = \left(\frac{\rho_{l,w}-\rho_{v,\infty}}{\rho_{l,s}}\right)^{1/2}\left(\frac{\rho_{v,s}-\rho_{v,\infty}}{\rho_{v,\infty}}\right)^{-1/2} W_{xl,s} \tag{29}$$

$$W_{yv,s} = -0.25\frac{\rho_{l,s}}{\rho_{v,s}}\left(\frac{\nu_{l,s}}{\nu_{v,\infty}}\right)^{1/2}\left(\frac{\rho_{l,w}-\rho_{v,\infty}}{\rho_{l,s}}\right)^{1/4} \times\left(\frac{\rho_{v,s}-\rho_{v,\infty}}{\rho_{v,\infty}}\right)^{-1/4}(\eta_{l\delta}W_{xl,s}-4W_{yl,s}) \tag{30}$$

$$\left(\frac{dW_{xv}}{d\eta_v}\right)_{\eta_v=0} = \frac{\mu_{l,s}}{\mu_{v,s}}\left(\frac{\nu_{v,s}}{\nu_{l,s}}\right)^{1/2}\left(\frac{\rho_{l,w}-\rho_{v,\infty}}{\rho_{l,s}}\right)^{3/4} \times\left(\frac{\rho_{v,s}}{\rho_{v,\infty}}-1\right)^{-3/4}\times\left(\frac{dW_{xl}}{d\eta_l}\right)_{\eta_l=\eta_{l,\delta}} \tag{31}$$

$$\left(\frac{d\theta_v}{d\eta_v}\right)_{\eta_v=0} = \frac{\left(\frac{\rho_{l,w}-\rho_{v,\infty}}{\rho_{l,s}}\right)^{1/4}\left(-h_{fg}\mu_{l,s}(\eta_{v\delta}W_{xv,s}-4W_{yv,s})+\lambda_{l,s}(T_w-T_s)\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=\eta_{l\delta}}\right)}{\left(\frac{\rho_{v,s}}{\rho_{v,\infty}}-1\right)^{1/4}\left(\frac{\nu_{l,s}}{\nu_{v,\infty}}\right)^{1/2}\lambda_{v,s}(T_s-T_\infty)} \tag{32}$$

$$\theta_l = 0 \tag{33}$$

$$\theta_v = 1 \tag{34}$$

$$\eta_v \to \infty: \quad W_{xv}\to 0 \quad \theta_v \to 0. \tag{35}$$

## 4. NUMERICAL SOLUTIONS

The treatment of variable thermophysical properties for both liquid and vapour must be done for solving the ordinary differential equations with the boundary condition equations as follows:

*For liquid film*, the polynomial method, we suggested in ref. [15] the treatment of variable thermophysical properties will be used for treating the film condensation of saturated vapour in [13]. The corresponding predictive expressions for density $\rho_l$, thermal conductivity $\lambda_l$ of water are, respectively

$$\rho_l = -4.48\times10^{-3}t^2+999.9 \tag{36}$$

$$\lambda_l = -8.01\times10^{-6}t^2+1.94\times10^{-3}t+0.563 \tag{37}$$

with the corresponding predictive expression for absolute viscosity $\mu_l$ of water [16]

$$\mu_l = \exp\left[-1.6-\frac{1150}{T}+\left(\frac{690}{T}\right)^2\right]\times10^{-3}. \tag{38}$$

Hence, the thermophysical property factors

$$\frac{1}{\rho_l}\frac{d\rho_l}{d\eta_l},\quad \frac{1}{\mu_l}\frac{d\mu_l}{d\eta_l},\quad \frac{1}{\lambda_l}\frac{d\lambda_l}{d\eta_l},\quad \frac{\mu_{l,s}}{\mu_l},\quad \frac{\nu_{l,s}}{\nu_l}$$

and

$$Pr_{l,s}\frac{\rho_l}{\rho_{l,s}}\frac{\lambda_{l,s}}{\lambda_l}$$

in the governing ordinary differential equations of condensate water film, equations (22)–(24), can be transformed into the following ones, respectively, as:

$$\frac{1}{\rho_l}\frac{d\rho_l}{d\eta_l} = \left[(-2\times4.48\times10^{-3}t)(t_w-t_s)\frac{d\theta_l}{d\eta_l}\right] \times(-4.48\times10^{-3}t^2+999.9)^{-1} \tag{39}$$

$$\frac{1}{\mu_l}\frac{d\mu_l}{d\eta_l} = \left(\frac{1150}{T^2}-2\times\frac{690^2}{T^3}\right)(t_w-t_s)\frac{d\theta_l}{d\eta_l} \tag{40}$$

$$\frac{1}{\lambda_l}\frac{d\lambda_l}{d\eta_l} = \times\left[(-2\times8.01\times10^{-6}t+1.94\times10^{-3})(t_w-t_s)\frac{d\theta_l}{d\eta_l}\right] \times(-8.01\times10^{-6}t^2+1.94\times10^{-3}t+0.562)^{-1} \tag{41}$$

$$\frac{\mu_{l,s}}{\mu_l} = \exp\left(1150\left(\frac{1}{T}-\frac{1}{T_s}\right)+690^2\left(\frac{1}{T_s}-\frac{1}{T}\right)\right) \tag{42}$$

$$\frac{\nu_{l,s}}{\nu_l} = \frac{\mu_{l,s}}{\mu_l}\frac{\rho_l}{\rho_{l,s}} = \exp\left(1150\left(\frac{1}{T}-\frac{1}{T_s}\right) +690^2\left(\frac{1}{T_s}-\frac{1}{T}\right)\right)\frac{-4.48\times10^{-3}t^2+999.9}{-4.48\times10^{-3}t_s^2+999.9} \tag{43}$$

$$Pr_{l,s}\frac{\rho_l}{\rho_{l,s}}\frac{\lambda_{l,s}}{\lambda_l} = Pr_{l,s}\left(\frac{-4.48\times10^{-3}t^2+999.9}{-4.48\times10^{-3}t_s^2+999.9}\right) \times\left(\frac{-8.01\times10^{-6}t_s^2+1.94\times10^{-3}t_s+0.563}{-8.01\times10^{-6}t^2+1.94\times10^{-3}t+0.563}\right) \tag{44}$$

where

$$t = (t_w-t_\infty)\theta_l+t_\infty. \tag{45}$$

*For the vapour*, the temperature parameter method, developed in [14], will be used. The viscosity and thermal conductivity of gases are expressed as $\mu_v \approx T^{n_\mu}$ and $\lambda_v \approx T^{n_\lambda}$, respectively, and the density of gases is regarded as $\rho_v \approx 1/T$. If the thermodynamic temperature of the gas far away from the wall, $T_\infty$, is taken as the reference temperature, there will be

$$\frac{\mu_v}{\mu_{v,\infty}} = \left(\frac{T}{T_\infty}\right)^{n_\mu} \tag{46}$$

$$\frac{\lambda_v}{\lambda_{v,\infty}} = \left(\frac{T}{T_\infty}\right)^{n_\lambda} \tag{47}$$

$$\frac{\rho_v}{\rho_{v,\infty}} = \left(\frac{T}{T_\infty}\right)^{-1} \tag{48}$$

while the change of kinematic viscosity at constant pressure can be expressed as

$$\frac{\nu_v}{\nu_{v,\infty}} = \left(\frac{T}{T_\infty}\right)^{n_\mu+1}. \tag{49}$$

Then, the thermophysical property factors

$$\frac{1}{\rho_v}\frac{d\rho_v}{d\eta_v},\quad \frac{1}{\mu_v}\frac{d\mu_v}{d\eta_v},\quad \frac{1}{\lambda_v}\frac{d\lambda_v}{d\eta_v},\quad \frac{\mu_{v,\infty}}{\mu_v},\quad \frac{\nu_{v,\infty}}{\nu_v}$$

and

$$\frac{\rho_v-\rho_{v,\infty}}{\rho_{v,s}-\rho_{v,\infty}}$$

in the governing ordinary differential equations of vapour film, equations (25)–(27), can be transformed, respectively, as below:

$$\frac{1}{\rho_v}\frac{d\rho_v}{d\eta_v} = -\frac{(T_s/T_\infty-1)\,d\theta_v/d\eta_v}{(T_s/T_\infty-1)\theta_v+1} \tag{50}$$

$$\frac{1}{\mu_v}\frac{d\mu_v}{d\eta_v} = \frac{n_\mu(T_s/T_\infty-1)\,d\theta_v/d\eta_v}{(T_s/T_\infty-1)\theta_v+1} \tag{51}$$

$$\frac{1}{\lambda_v}\frac{d\lambda_v}{d\eta_v} = \frac{n_\lambda(T_s/T_\infty-1)\,d\theta_v/d\eta_v}{(T_s/T_\infty-1)\theta_v+1} \tag{52}$$

$$\frac{\mu_{v,\infty}}{\mu_v} = [(T_s/T_\infty-1)\theta_v+1]^{-n_\mu} \tag{53}$$

$$\frac{\nu_{v,\infty}}{\nu_v} = [(T_s/T_\infty-1)\theta_v+1]^{-(n_\mu+1)}. \tag{54}$$

The calculation procedure for the two-phase boundary layer problem of superheated film condensation is carried out numerically by two steps, as developed in [13]. First, the solutions of equations (22)–(24) are assumed to be without drag of vapour or shear force at the liquid–vapour interface, and then, the boundary condition, e.g. equation (31), could be changed into

$$\left(\frac{dW_{xl}}{d\eta_l}\right)_{\eta_l=\eta_{l,\delta}} = 0. \tag{55}$$

Equations (28), (33) and (55) are taken as the two-point boundary conditions of equations (22)–(24) for liquid film, and are solved by a Shooting Method [17]. The second step is carried out as a three-point boundary problem for coupling equations of liquid film with equations for vapour. The boundary values $W_{xv,s}$ and $W_{yv,s}$ are found by equations (29) and (30), respectively, and then equations (25)–(27) are calculated with the boundary conditions (34) and (35).

Table 1. The thermophysical property values for water and steam at saturated temperature

| Phase | Water | Steam |
|---|---|---|
| $t$ [°C] | 100 | 100 |
| $c_p$ [J kg$^{-1}$ K$^{-1}$] | 4216 | |
| $h_{fg}$ [kJ kg$^{-1}$] | | 2257.3 |
| $Pr$ | 1.76 | 1 |
| $\rho$ [kg m$^{-3}$] | 958.1 | 0.9574 |
| $\mu$ [kg m$^{-1}$ s$^{-1}$] | $282.2\times10^{-6}$ | $12.28\times10^{-6}$ |
| $\nu$ [m$^2$ s$^{-1}$] | $0.294\times10^{-6}$ | $20.55\times10^{-6}$ |
| $\lambda$ [W m$^{-1}$ K$^{-1}$] | 0.677 | 0.02478 |

Table 2. The values of water density at different wall temperatures

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $t$ [°C] | 0 | 20 | 40 | 60 | 80 | 95 | 99.9 |
| $\rho$ [kg m$^{-3}$] | 999.8 | 998.3 | 992.3 | 983.2 | 971.4 | 961.7 | 958.1 |

Adjudgment equations (31) and (32) are used for checking convergence of the solutions; the calculation is iterated with an appropriate change of the values $W_{xl,s}$ and $\eta_{l\delta}$. In each iteration, the calculations of equations (22)–(24) for liquid film and equations (25)–(27) for vapour are made successively by the Shooting Method.

All thermophysical properties for water and steam used in the calculations come from ref. [18]. For convenience, some special values of the thermophysical properties are listed in Tables 1 and 2.

The numerical calculations have been carried out for wall subcooled temperatures $\Delta t_w(=t_s-t_w) = 0.1$, 5, 20, 40, 60, 80, 100°C and for steam superheated temperatures $\Delta t_\infty(=t_\infty-t_s) = 0$, 27, 127, 227, 327, 427°C. The calculated results for the velocity and temperature fields on the medium in the liquid and vapour films are plotted in Figs. 2 and 3, respectively.

## 5. HEAT TRANSFER ANALYSIS

As reported in ref. [13] for film condensation of saturated vapour, the local heat transfer rate from wall to condensate film can be described as

$$q_x = -\lambda_{l,w}(T_w-T_s)\left(\frac{1}{4}Gr_{xl,s}\right)^{1/4} x^{-1}\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}. \tag{56}$$

Then, the local heat transfer coefficient will be

$$\alpha_x = -\lambda_{l,w}\left(\frac{1}{4}Gr_{xl,s}\right)^{1/4} x^{-1}\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0} \tag{57}$$

or, the local Nusselt number, defined as

$$Nu_{xl,w} = \frac{\alpha_x x}{\lambda_{l,w}},$$

can be expressed by

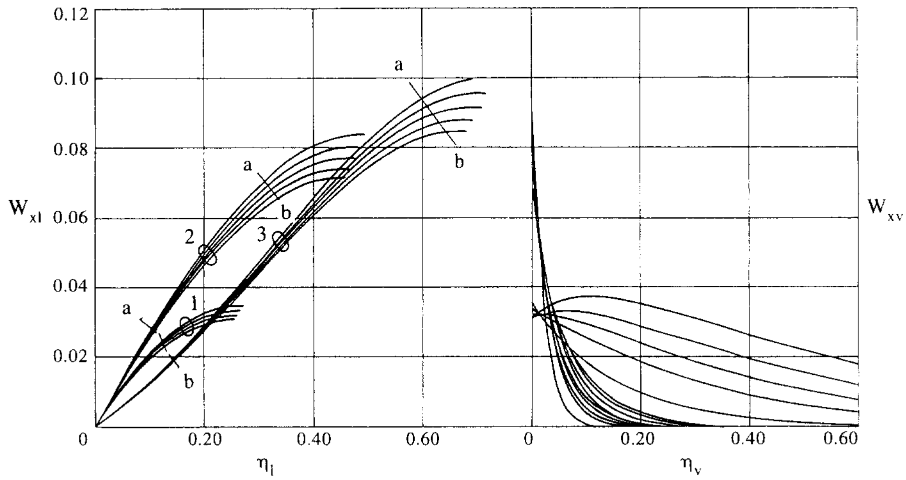


Fig. 2. Velocity profiles for film condensation of superheated steam.

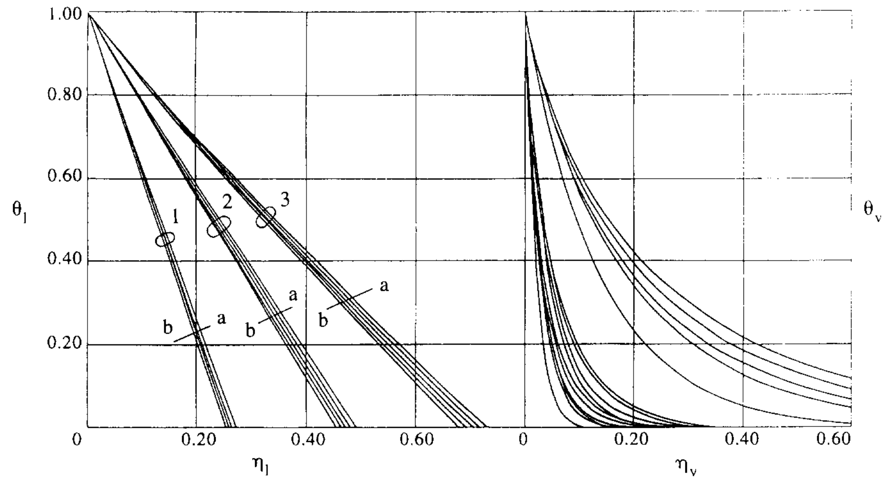


Fig. 3. Temperature profiles for film condensation of superheated steam.

$$Nu_{xl,w} = -\left(\frac{1}{4}Gr_{xl,s}\right)^{1/4}\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}. \quad (58)$$

The mean heat transfer coefficient, $\bar{\alpha}_x$, and the mean Nusselt number, defined as

$$\overline{Nu}_{xl,w} = \frac{\bar{\alpha}_x x}{\lambda_{l,w}},$$

will be respectively

$$\bar{\alpha}_x = \tfrac{4}{3}\alpha_x \quad (59)$$

$$\overline{Nu}_{xl,w} = \tfrac{4}{3}Nu_{xl,w}. \quad (60)$$

The numerical solutions

$$\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}$$

for film condensation of superheated steam at the corresponding temperature conditions, $\Delta t_w$ and $\Delta t_\infty$, are summarized in Table 3, and plotted in Fig. 4. According to these numerical solutions and, with a curve matching method, we obtain the following correlation for prediction of

$$\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}$$

for the laminar film condensation of superheated steam with $5°C \leqslant \Delta t_w \leqslant 100°C$

$$-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0} = \left[-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}\right]_{\Delta t_\infty=0} + a\cdot\frac{\Delta t_\infty}{t_s} \quad (61)$$

where

$$\left[-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}\right]_{\Delta t_\infty=0}$$

is referred to as the dimensionless temperature gradient of the film condensation of saturated steam on the

Table 3. Values for $-(d\theta/d\eta_l)_{\eta_l=0}$: (1) numerical solution and (2) predicted by equations (61)–(63)

| $\Delta t_\infty$ | | $\Delta t_w$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | 5 | 20 | 40 | 60 | 80 | 100 |
| 0 | (1) | 3.6707 | 2.5397 | 2.0789 | 1.8324 | 1.6679 | 1.5511 |
| | (2) | 3.6596 | 2.5451 | 2.0924 | 1.8475 | 1.6971 | 1.55 |
| 27 | (1) | 3.6913 | 2.5532 | 2.0891 | 1.8411 | 1.6759 | 1.5581 |
| | (2) | 3.6770 | 2.5592 | 2.1032 | 1.8561 | 1.6866 | 1.5576 |
| 127 | (1) | 3.7712 | 2.6054 | 2.1299 | 1.8758 | 1.7066 | 1.5861 |
| | (2) | 3.7416 | 2.6117 | 2.1431 | 1.8878 | 1.7145 | 1.5858 |
| 227 | (1) | 3.8478 | 2.6551 | 2.1688 | 1.9091 | 1.7362 | 1.6131 |
| | (2) | 3.8062 | 2.6641 | 2.1831 | 1.9196 | 1.7423 | 1.6140 |
| 327 | (1) | 3.9245 | 2.7046 | 2.2079 | 1.9425 | 1.7659 | 1.6402 |
| | (2) | 3.8708 | 2.7166 | 2.2230 | 1.9513 | 1.7701 | 1.6422 |
| 427 | (1) | 3.9555 | 2.7512 | 2.2449 | 1.9738 | 1.7938 | 1.6656 |
| | (2) | 3.9354 | 2.7690 | 2.2630 | 1.9831 | 1.7980 | 1.6698 |

wall. According to the correlation proposed in ref. [13] and by introducing the temperature ratio $\Delta t_w/t_s$, the correlation of

$$\left[-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}\right]_{\Delta t_\infty=0}$$

will be

$$\left[-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}\right]_{\Delta t_\infty=0}=\frac{1.74-0.19\dfrac{\Delta t_w}{t_s}}{\left(\dfrac{\Delta t_w}{t_s}\right)^{1/4}} \quad (62)$$

the coefficient $a$ is expressed as follows for the film condensation of superheated steam:

$$a=10^{-2}\times\left(6.92-9.45\frac{\Delta t_w}{t_s}+5.35\left(\frac{\Delta t_w}{t_s}\right)^2\right). \quad (63)$$

This follows that the results in [13] would be a special case with $\Delta t_\infty = 0$.

The predicted values for

$$-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}$$

from equations (61)–(63) are summarized and compared with the numerical results in Table 3.

## 6. MASS TRANSFER ANALYSIS

With the same derivation as shown in ref. [13] for the film condensation of saturated vapour, the mass flow rate of the film condensate of superheated vapour, through an area $1 \times x$ with unit width starting from the bottom of the plate on the liquid–vapour interface, $G_x$ should be expressed for the film condensation of superheated vapour as:

$$\frac{G_x}{\mu_{l,s}}=\frac{4}{3}\left(\frac{1}{4}Gr_{xl,s}\right)^{1/4}(\eta_{l\delta}W_{xl,s}-W_{yl,s}). \quad (64)$$

It follows that $G_x$ depends on the defined local Grashof number $Gr_{xl,s}$, absolute viscosity $\mu_{l,s}$ and mass

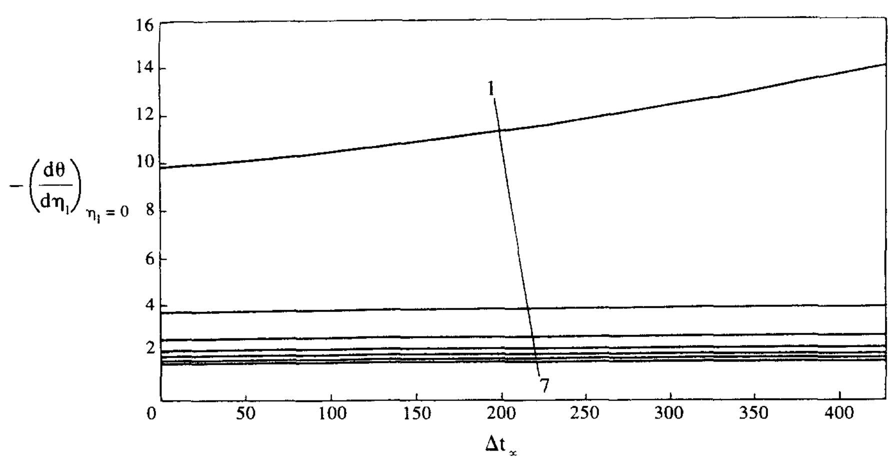


Fig. 4. Numerical solutions of $-(d\theta/d\eta_l)_{\eta_l=0}$ with $\Delta t_\infty$ and $\Delta t_w$.

Table 4. Numerical solution for dimensionless boundary layer thickness $\eta_{\delta_l}$

| $\Delta t_\infty$ [°C] | $\Delta t_w$ [°C] 0.1 | 5 | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|---|
| 0 | 0.1017 | 0.27307 | 0.39844 | 0.4958 | 0.577 | 0.65545 | 0.73561 |
| 27 | 0.10032 | 0.27155 | 0.39631 | 0.49331 | 0.57416 | 0.65217 | 0.73207 |
| 127 | 0.09368 | 0.26577 | 0.38826 | 0.4836 | 0.5631 | 0.63981 | 0.71838 |
| 227 | 0.08623 | 0.26047 | 0.3809 | 0.4747 | 0.55291 | 0.6284 | 0.70572 |
| 327 | 0.07848 | 0.25536 | 0.37384 | 0.46611 | 0.5431 | 0.61739 | 0.6935 |
| 427 | 0.07078 | 0.25335 | 0.36745 | 0.45825 | 0.5342 | 0.6074 | 0.68243 |

Table 5. Numerical solutions of $W_{xl,\delta}$

| $\Delta t_\infty$ [°C] | $\Delta t_w$ [°C] 0.1 | 5 | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|---|
| 0 | 0.00513 | 0.03546 | 0.06619 | 0.0851 | 0.09463 | 0.09925 | 0.10111 |
| 27 | 0.00504 | 0.03509 | 0.06544 | 0.08413 | 0.09353 | 0.09805 | 0.0999 |
| 127 | 0.00450 | 0.03369 | 0.06270 | 0.08053 | 0.08945 | 0.09373 | 0.09547 |
| 227 | 0.00391 | 0.03245 | 0.06027 | 0.07733 | 0.08585 | 0.08992 | 0.09156 |
| 327 | 0.00333 | 0.03130 | 0.05801 | 0.07435 | 0.0825 | 0.08637 | 0.08794 |
| 427 | 0.00280 | 0.03095 | 0.05603 | 0.07171 | 0.07957 | 0.08328 | 0.08478 |

Table 6. Numerical solutions of $-W_{yl,\delta}$

| $\Delta t_\infty$ [°C] | $\Delta t_w$ [°C] 0.1 | 5 | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|---|
| 0 | 0.00013 | 0.002416 | 0.0065 | 0.010085 | 0.01237 | 0.01355 | 0.01371 |
| 27 | 0.000126 | 0.002377 | 0.006398 | 0.009928 | 0.012176 | 0.01334 | 0.013509 |
| 127 | 0.000104 | 0.002231 | 0.006012 | 0.00934 | 0.011466 | 0.01257 | 0.012747 |
| 227 | 0.000082 | 0.002103 | 0.005674 | 0.008823 | 0.01084 | 0.0119 | 0.01207 |
| 327 | 0.000063 | 0.001985 | 0.005363 | 0.008344 | 0.01026 | 0.011268 | 0.011443 |
| 427 | 0.000047 | 0.001943 | 0.005092 | 0.007923 | 0.009754 | 0.01072 | 0.0109 |

Table 7. $\eta_{l\delta} \cdot W_{xl,\delta} - 4W_{yl,\delta}$ with different $\Delta t_\infty$ and $\Delta t_w$: (1) numerical solution and (2) predicted from equations (65)–(67)

| $\Delta t_\infty$ [°C] | | $\Delta t_w$ [°C] 5 | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|---|
| 0 | (1) | 0.019347 | 0.052373 | 0.082533 | 0.104082 | 0.119253 | 0.129218 |
| | (2) | 0.019366 | 0.052218 | 0.082085 | 0.103487 | 0.118764 | 0.129 |
| 27 | (1) | 0.019037 | 0.051526 | 0.081214 | 0.102402 | 0.117305 | 0.127170 |
| | (2) | 0.019132 | 0.051552 | 0.080959 | 0.102029 | 0.117104 | 0.127268 |
| 127 | (1) | 0.017878 | 0.048391 | 0.076302 | 0.096233 | 0.110249 | 0.119572 |
| | (2) | 0.018264 | 0.049089 | 0.076787 | 0.096629 | 0.110956 | 0.120852 |
| 227 | (1) | 0.016865 | 0.045654 | 0.072001 | 0.090827 | 0.104103 | 0.112896 |
| | (2) | 0.017396 | 0.046625 | 0.072616 | 0.09123 | 0.104809 | 0.114437 |
| 327 | (1) | 0.015932 | 0.043139 | 0.068030 | 0.085846 | 0.098396 | 0.106758 |
| | (2) | 0.016529 | 0.044162 | 0.068444 | 0.08583 | 0.098661 | 0.108021 |
| 427 | (1) | 0.015612 | 0.040956 | 0.064551 | 0.081520 | 0.093464 | 0.101456 |
| | (2) | 0.015661 | 0.041698 | 0.064273 | 0.080431 | 0.092514 | 0.101605 |

flow rate parameter of the film condensation of superheated vapour $(\eta_{l\delta} W_{xl,s} - W_{yl,s})$.

The numerical solutions $\eta_{l\delta}$, $W_{xl,s}$, $W_{yl,s}$ as well as $(\eta_{l\delta} W_{xl,s} - W_{yl,s})$, $\Delta t_\infty$ and $\Delta t_w$ are listed in Tables 4–7, and plotted in Figs. 5–8, respectively.

According to the corresponding numerical solutions and by the curve-matching method, the correlated expressions for the mass flow rate parameter $(\eta_{l\delta} W_{xl,s} - W_{yl,s})$ of the film condensation of superheated steam with $5°C \leqslant \Delta t_w \leqslant 100°C$ is obtained as

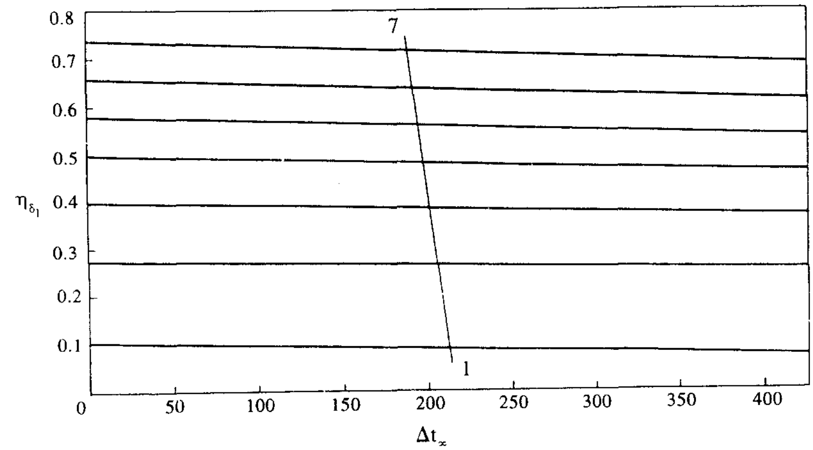


Fig. 5. Numerical solution for $\eta_{\delta_1}$ with $\Delta t_\infty$ and $\Delta t_w$.

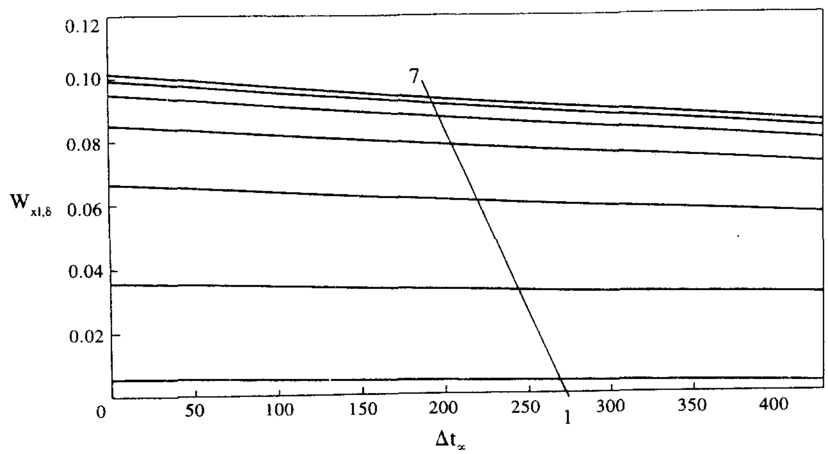


Fig. 6. Numerical results of $W_{x1,\delta}$ with different $\Delta t_\infty$ and $\Delta t_s$.

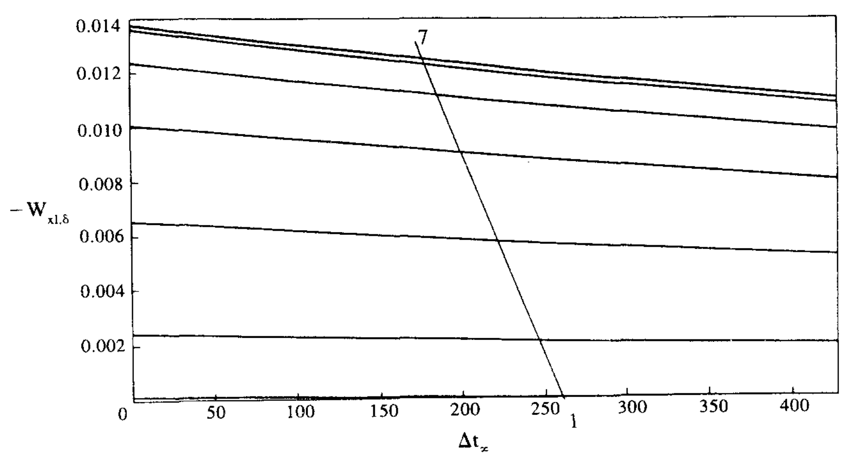


Fig. 7. Numerical results of $-W_{y1,\delta}$ with different $\Delta t_\infty$ and $\Delta t_s$.

$$\eta_{1\delta}\cdot W_{x1,\delta}-4W_{y1,\delta}=(\eta_{1\delta}\cdot W_{x1,\delta}-4W_{y1,\delta})_{\Delta t_\infty=0}-b\frac{\Delta t_\infty}{t_s} \quad (65)$$

where, by introducing the temperature ratio $\Delta t_w/t_s$, the mass flow rate parameter for the film condensation of saturated steam, $(\eta_{1\delta}\cdot W_{x1,\delta}-4W_{y1,\delta})_{\Delta t_\infty=0}$, and $b$ can be expressed, respectively, as:

$$(\eta_{1\delta}\cdot W_{x1,\delta}-4W_{y1,\delta})_{\Delta t\infty=0}=\left(0.186-0.057\frac{\Delta t_w}{t_s}\right)\left(\frac{\Delta t_w}{t_s}\right)^{3/4} \quad (66)$$

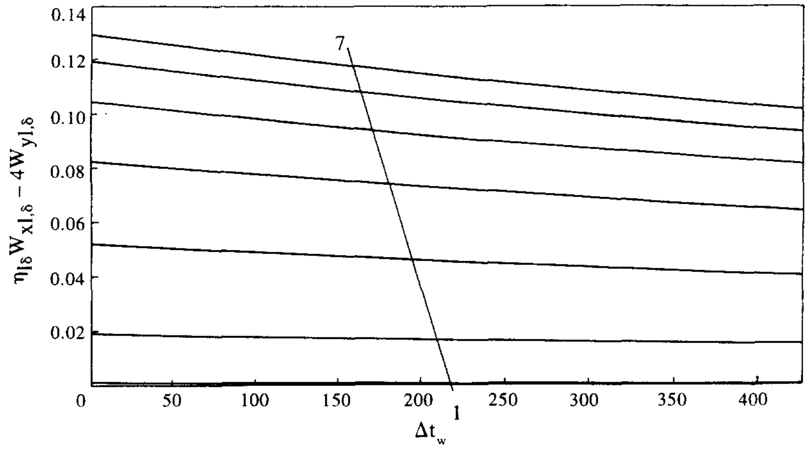


Fig. 8. Numerical results of $\eta_{l\delta}W_{xl,\delta}-4W_{yl,\delta}$ with different $\Delta t_\infty$ and $\Delta t_w$.

$$b = 10^{-4}\times\left[2.756+121.4\frac{\Delta t_w}{t_s}-60\left(\frac{\Delta t_w}{t_s}\right)^2\right]. \tag{67}$$

The predicted $(\eta_{l\delta}W_{xl,s}-W_{yl,s})$, from equations (65)–(67) for the laminar film condensation of superheated steam, are summarized and compared with the numerical results in Table 7.

## 7. CONCLUDING REMARKS

In this present work, we deal with the theory of laminar film condensation of superheated vapour on a vertical flat plate at atmospheric pressure with consideration of the various factors including the variable thermophysical properties. The major objective is to obtain theoretically rigorous expressions for the prediction of heat and mass transfer. The study presented here is an extension of former studies and demonstrated the following points.

(1) An advanced similarity transformation approach, dimensionless velocity component method, is used to transform the system of partial differential equations associated with the two-phase boundary problem into a system of dimensionless ordinary equations. This transformation has an obvious advantage over the corresponding Falkner–Skan transformation [19].

(2) The system of ordinary differential equations and its related boundary conditions is computed by a successively iterative procedure for the numerical solutions of the three-point boundary problem. Meanwhile, the following theoretical correlations for Nusselt number $Nu_{xl,w}$ and mass flow rate $G_x$ are derived for the laminar flow condensation of superheated vapour as:

$$Nu_{xl,w} = -\left(\frac{1}{4}Gr_{xl,s}\right)^{1/4}\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0} \tag{58}$$

$$\frac{G_x}{\mu_{l,s}} = \frac{4}{3}\left(\frac{1}{4}Gr_{xl,s}\right)^{1/4}(\eta_{l\delta}W_{xl,s}-W_{yl,s}). \tag{64}$$

Both the local Nusselt number $Nu_{xl,w}$ and mass flow rate $G_x$ of the film condensation of superheated vapour are proportional to the local Grashof number $Gr_{xl,s}$. In addition, $Nu_{xl,w}$ is also proportional to the temperature gradient on the wall,

$$\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0},$$

and $Gr_{xl,s}$ is proportional to the mass flow rate parameter $(\eta_{l\delta}W_{xl,s}-W_{yl,s})$.

(3) From the numerical solution of the film condensation of superheated steam and by the curve-matching method, the practical simple correlations for predicting

$$\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}$$

and $(\eta_{l\delta}W_{xl,s}-W_{yl,s})$ are, respectively, obtained for $5°C \leqslant \Delta t_w \leqslant 100°C$, as:

$$\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0} = \left[-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}\right]_{\Delta t_w=0} + a\frac{\Delta t_\infty}{t_s} \tag{61}$$

$$\eta_{l\delta}W_{xl,s}-W_{yl,s} = (\eta_{l\delta}W_{xl,s}-W_{yl,s})_{\Delta t_w=0} - b\frac{\Delta t_\infty}{t_s} \tag{68}$$

where

$$\left[-\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l=0}\right]_{\Delta t_\infty=0} = \frac{1.74-0.19\dfrac{\Delta t_w}{t_s}}{\left(\dfrac{\Delta t_w}{t_s}\right)^{1/4}} \tag{62}$$

$$a = 10^{-2}\times\left[6.92-9.45\frac{\Delta t_w}{t_s}+5.35\left(\frac{\Delta t_w}{t_s}\right)^2\right] \tag{63}$$

$$(\eta_{l\delta} \cdot W_{xl,\delta} - 4W_{yl,\delta})_{\Delta t_\infty = 0}$$

$$= \left(0.186 - 0.057\frac{\Delta t_w}{t_s}\right)\left(\frac{\Delta t_w}{t_s}\right)^{3/4} \quad (66)$$

$$b = 10^{-4} \times \left[2.756 + 121.4\frac{\Delta t_w}{t_s} - 60\left(\frac{\Delta t_w}{t_s}\right)^2\right] \quad (67)$$

$$Gr_{xl,s} = \frac{g(\rho_{l,w} - \rho_{v,\infty})x^3}{\nu_{l,s}^2 \rho_{l,s}}. \quad (14)$$

(4) Regarding the effect of subcooled temperature $\Delta t_w$ and superheated temperature $\Delta t_\infty$ on the film condensation of superheated steam, we obtain the following main conclusions:

the subcooled temperature on the wall, $\Delta t_w$, and the superheated temperature of steam, $\Delta t_\infty$, dominate the dimensionless temperature gradient

$$\left(\frac{d\theta_l}{d\eta_l}\right)_{\eta_l = 0},$$

(Table 3 and Fig. 4), as well as the mass flow rate parameter $(\eta_{l\delta} \cdot W_{xl,\delta} - 4W_{yl,\delta})$ (see also Table 7 and Fig. 8). These demonstrate that $\Delta t_w$ and $\Delta t_\infty$ affect the heat transfer coefficient and mass flow rate for the film condensation of superheated steam.

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